THE EFFECT OF HYDROSTATIC PRESSURE ON THE MAGNETIC PERMEABILITY OF IRON, COBALT AND NICKEL.

By Chi-Sun Yeh.

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INTRODUCTION.

THE various phenomena of magnetostriction together with their inverse effects, namely, the effects of stress on magnetization, have been known for a long time. In particular, Nagaoka and Honda (1) studied the effect of hydrostatic pressure on the magnetization of iron and nickel, and later on, Miss Frisbie (2) repeated the work for iron. The highest pressure used by Nagaoka and Honda was about 300 atmospheres; that by Miss Frisbie about 1000 atmospheres. For iron, while Nagaoka and Honda obtained only a decrease of magnetization by pressure, Miss Frisbie obtained an increase in low fields and a decrease in high fields, the range of field covered by Miss Frisbie being within that covered by Nagaoka and Honda. Besides this disagreement in results, the data presented by these authors are not

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sufficient for giving a comprehensive view of the pressure effect on magnetization.

The present research was undertaken because we thought that the subject could now be attacked more comprehensively and also to better advantage. There are three reasons for this: (1) the work of Professor Bridgman has enabled us to extend the pressure to a much wider range than covered by earlier workers; (2) the ferromagnetic metals can now be obtained in a much purer condition than those used by the earlier experimenters; (3) the results may stimulate interest in developing magnetic theories along modern lines.

EXPERIMENTAL METHOD IN GENERAL.

In order to avoid end effects, toroidal specimens were used. The method of balancing the magnetic deflection against that due to another specimen, made as nearly similar as possible to the pressure sample, was rejected at the very beginning of the work, since it cannot give reliable results on account of the great difficulty of getting exactly similar specimens. The *increase* of magnetization of iron under pressure for H about 5 obtained by Miss Frisbie might be easily explained by this error. (This wrong effect might also be due to imperfect demagnetization as we will see later.) Instead of the method of Miss Frisbie, we balanced the deflection due to the specimen under pressure against that due to a mutual air inductance.

Two kinds of runs were made, one at a constant magnetizing field, and the other at a constant pressure. They will be described separately.

Constant Field Runs.—For these runs, the ballistic deflection due to the magnetic specimen was nearly completely balanced against that due to a mutual inductance, leaving however a residual reversal deflection greater than the total change of reversal deflection at 12000 kg./cm². This deflection usually amounted to several centimeters. It was not possible to balance the magnetic deflection exactly when there was no pressure for the reason that the two parts of the magnetic reversal deflection, one obtained on breaking the circuit and the other on making the circuit in the opposite direction, were unequal because of hysteresis. The result was that during the initial stages of the reversal there was a slight deflection in a direction opposite from the final deflection. The effect probably could be eliminated by increasing the period of the galvanometer or by increasing the rapidity of reversal. Too rapid reversals are, however, not desirable on account of the dif-

ficulty of insuring uniform operation. The final scheme adopted was to minimize the initial reverse deflection as much as possible by using two mutual inductances of different values, such that one of them is in circuit when the magnetizing current goes one way, while the other is in circuit when the magnetizing current goes the other way. This arrangement was made possible by using a reversing switch of eight poles. The mutual inductances were made of solenoids with sliding secondary coils inside the primary. They were specially made to give the large variation demanded in this work.

From the direct observations, the differences of residual reversal deflections were calculated. When these are divided by the total reversal deflection of the specimen alone (without the compensating mutual inductance) under no pressure, we obtain the percentage change of permeability or induction. This is practically the same as the percentage change of susceptibility or magnetization for the ferromagnetic substances in the range of fields we are using. These percentage changes have to be corrected for the change of dimensions under pressure, because in the calculation of B and H from experimental data, the cross section and circumferential length of the ring come into consideration. When we consider the decrease of area under pressure, twice the absolute value of linear compressibility must be added to the uncorrected pressure coefficient. When we consider further the circumferential linear contraction and the consequent higher value of H we must subtract the absolute value of the linear compressibility, making a total additive correction of the absolute value of the linear compressibility. The magnitude of this correction is .02% for each 1000 kg./cm². It should be deducted from the numerical value of the observed percentage change, when the pressure coefficient of magnetization is negative; it should be added when positive.

When the corrected percentage changes were plotted against pressure, the first few runs showed a flat region in the curve plotting $\Delta B/B_0$ against pressure or else showed a curve of continually increasing slope. Further, the results were often irregular. The cause of the trouble was later traced to imperfect demagnetization. The final scheme adopted was to demagnetize the specimen completely before every change of pressure. When this was done, the percentage change became linear with pressure within the limits of experimental error, and the results also became reproducible.

A word should be said about the meaning of the reversal deflection as measured here. As is well known, the first reversal deflection is

greater than the later ones.   What was measured here was the constant reversal deflection obtained after several reversals, which can be determined with greater accuracy than the initial reversal deflection.   The question of accuracy is of particular importance here because we are dealing with small differences.   The values of permeability and also of differential permeability therefore come out considerably lower than when measured by the step-by-step method.

In using the balanced reversal method, we have to be particularly careful about the effect of imperfect demagnetization.   For suppose we start with a specimen not completely demagnetized, then the reversal deflection under no pressure and low fields will be smaller than it ought to be.   If, now, the residual magnetization is a function of the pressure, the error in the residual deflection will vary with pressure so that the apparent percentage change produced by pressure may be complicated in character and entirely erroneous; it may even be of the wrong sign.   Under high fields an erroneous result is less likely.

In general, the effects of imperfect magnetization in iron and nickel are opposite, since their pressure coefficients are of opposite signs.

Constant pressure runs.—This type of run was to determine the percentage change of permeability as a function of the field.   It is less conducive to accurate measurement than the constant field run; but it gives a more comprehensive idea about the course of phenomena.   In this type of run, we simply obtain a normal magnetization curve under no pressure, and then one under high pressure.   This method gives reliable results if the pressure is sufficiently high.   The method of balancing the magnetic deflection against a mutual inductance deflection is here not used on account of the inconvenience of changing the mutual inductance adjustments before every increase of the magnetizing current.   Even if the balancing method were used, it would not give as accurate results as with the constant field runs.

Experimental Details.

The pressure apparatus designed by Professor Bridgman has been thoroughly described in his papers (3).   It suffices to say that the apparatus can hold pressure with practically no leak for the range covered in this paper, namely, 0–12000 kg./cm^2.   The measurement of pressure by the change of resistance of manganin wire under pressure has also been described in Professor Bridgman's papers (4).   By comparing with an absolute gauge he showed that the change of resistance of manganin is linear with pressure for the range covered here.

The pressure values reported below were obtained as follows: the total resistance of the gauge and the change of resistance under pressure were directly measured on the bridge. The pressure was then calculated with the coefficient 2.325×10^{-6} per kg/cm^2. The maximum error that may be introduced in the absolute value of pressure on account of the slight variation of the pressure coefficient for different specimens from same spool is about 2%. The error in the relative values of pressure is less than 1%.

The diameter of the hole of the cylinder containing the specimen was about 16 mm., and its external diameter about 8 times the diameter of the hole. These dimensions of the high pressure chamber necessitated the use of toroidal specimens having an outer diameter of about 16 mm. and an arm width of about 3.2 mm. The toroids were made into shape with a semi-circular cutter.

The specimen was mounted horizontally on a collar screwed to a three-terminal plug. The three terminals served for the secondary leads and one of the primary leads. The other primary lead was attached to the body of the plug. Through each hole passed a steel stem surrounded by packing which served for keeping the pressure as well as for insulating the stem from the main body of the plug. The perfection of the insulation was tested both on the bridge for measuring pressure and with the ballistic galvanometer, the criterion with the latter being that its zero should be exactly the same no matter whether the primary current was going one way or the other. No particular care was taken to compensate or minimize the earth's field, because the effective earth's field inside the pressure cylinder must be exceedingly small.

The primary winding was such that one ampere gave about 50 gausses. The total number of secondary turns ranged from 200 to 500. For the primary winding, enameled copper wire was used; the diameter of the bare wire was about .2 mm. The change of resistance of the primary by pressure does not affect our results, because the constancy of current was watched with a potentiometer. For the secondary winding, silk insulated copper wire was used, the diameter of the bare wire being about 0.76 mm. The total resistance of the secondary winding varied from 10 to 20 ohms. Since the total change of resistance of copper under 12000 kg./cm^2. is about 2% of the resistance under no pressure, the maximum change of resistance of the secondary that might affect our results is about .4 ohm. Since there are more than 9000 ohms in the secondary circuit, the correction to be applied is at most .005%. We need not consider this correction since we give results only to a hundredth of a per cent.

Before putting on the primary winding, the anchor ring was insulated with a thin layer of enamel, baked on in an oven at a temperature around 205° C. A single layer of the primary was wound all over the circumference of the ring, care being taken to keep the winding radial. The secondary was then wound outside of the primary, no care being taken to keep the winding radial; because it was not necessary. What enters the measurement of the induction is the projection in the direction of the field of the area through which the lines of force run. This projection is always the cross section of the arm of the ring. Winding was done by hand and the number of turns was counted at the same time.

The magnetizing current was kept constant by watching a potentiometer of very simple scheme, (see fig. 1) a standard cell being used

Standard Cell

Galvanometer

Fixed Resistance in Primary Circuit

Primary Circuit
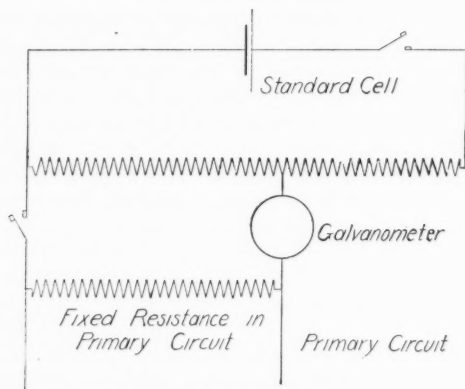
FIGURE 1.   Potentiometer.

directly in closed circuit in series with a 50,000 ohm resistance during balancing. The e.m.f. of the standard cell was compared with that of a better standard from time to time. It kept its value within one thousandth of a per cent.

The field, H, is given by the relation $H = (4/10)NI$ for a thin toroid, where I is the current in amperes and N the number of primary turns per cm. I was measured on the potentiometer. N was obtained by dividing the total number of primary turns by the mean circumference of the toroid. The largest uncertainty in the measurement of field lies in the fact that our toroids were not thin, the ratio of arm

width to mean diameter varying from 0.26 to 0.31. The error that may be introduced on this account is about 3% according to a calculation by Lloyd (5). Errors in measuring N and I are certainly much less than this.

The magnetic measurements were carried out with a ballistic galvanometer. It was used in the nearly critically damped condition (slightly over damped) with 9680 ohms in the circuit. The galvanometer (Cat. No. 2285–D Serial No. 73779) was obtained from Leeds and Northrup. Its free period is 28 sec., ballistic period, 6 sec. It was used at a distance of about 3 meters and gave a sensitivity of .0092 microcoulomb per millimeter.

The equation for calculating the induction, B, is

$$B = \frac{KRd}{nA}$$

where $d = \frac{1}{2}$ reversal deflection, n = total number of secondary turns, A = cross section area of the arm of the anchor ring, R = total resistance in the secondary circuit, K = galvanometer constant. Since R was kept constant, the calibration of the ballistic galvanometer amounts to finding the combination of constants $K \times R$. This was done by means of a mutual inductance standard, of the value 0.964 millihenries. The relation for calculating KR is the following:

$$K \times R = \frac{MI}{d_s}$$

where M = mutual inductance, d_s = deflection observed when we make or break a current of value I. The absolute value of $K \times R$ is only needed for the $B - H$ curves, not for the pressure coefficients of magnetization. Throughout the series of runs, the galvanometer constant was found to keep its value within .08%.

When the deflections became too large, a shunt of ratio 2.55 was used in parallel with the ballistic galvanometer. The shunt ratio was directly determined by comparing the deflections obtained with and without the shunt.

The electrical connections, except for minor conveniences, are sketched in fig. 1–3. With regard to the primary circuit, the reader will note that when the switch is closed one way, the current follows the path $AC(MI_1)$ D—toroid—EB; when reversed the path is $AC''(MI_2)$ $D'E$—toroid—$DE'B$. The exact equality of the two paths of the primary circuit as far as resistance is concerned was tested with the potentiometer. Outside of the ferromagnetic specimen, the cir-

cuits were free from iron, except the steel stems which serve as electric leads through the three-terminal plug. These steel stems can intro-
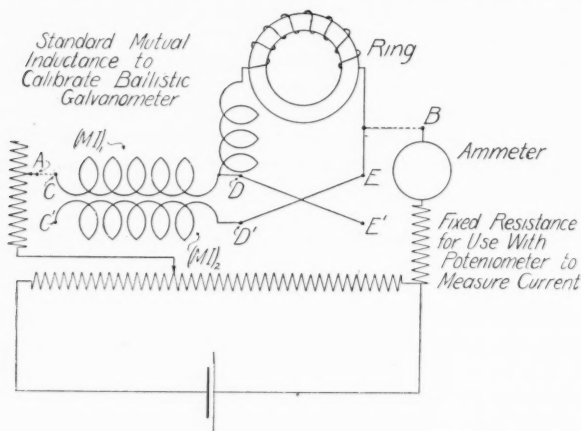

Fig 2   Primary Circuit

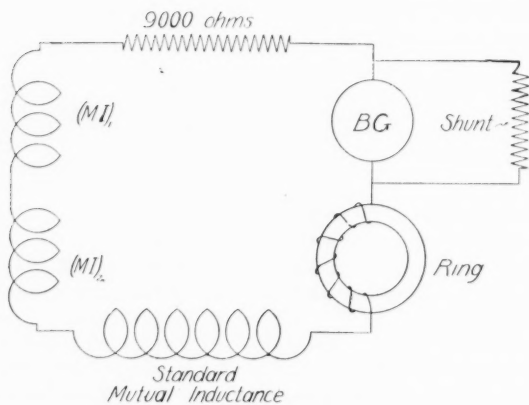
Standard
Mutual Inductance
Fig. 3   Secondary Circuit

duce no error in the magnetic measurement as long as there is no secondary turn linking with them.

The specimens were demagnetized by using a gradually decreasing alternating current (60 cycles), starting from a current higher than the maximum magnetizing current. The current was varied by varying the potential with a sliding resistance of tubular type.

The pressure cylinder containing the specimen was surrounded with a tank of water with a stirrer. The temperature of the bath was kept around 20° C. Since the temperature coefficient of magnetization around room temperature is small, the pressure coefficient could be measured without taking elaborate means to keep the temperature absolutely constant; the temperature of the bath was constant within a degree.

The heating effect of the magnetizing current is a factor that must be considered. With one ampere current, the temperature of the specimen is probably 1° higher than that of its surroundings. This is one of the reasons for not pushing the magnetizing current beyond two amperes.

THE MATERIALS

The specimens examined are described as follows:

Pure Iron.—The specimen was of French preparation, obtained from Professor Sauveur. It is known to contain 99.98% iron. It is

FIGURE 4.   Microphotograph of Pure Iron.

indeed very pure as shown by its microphotograph (fig. 4). The material had been previously annealed by imbedding in lime. The toroid, after being shaped, was annealed again by heating to 1000° C. in an electric furnace, followed by cooling in the furnace. The specimen, while annealing, was imbedded in a large quantity of iron dust

to prevent oxidation.   Microphotographic examination of the toroid after annealing shows that the material remained very pure.

Slightly Carbonized Iron.—This specimen started its career as a very pure specimen of iron, obtained from the Bureau of Standards, having the following analysis: C .005%, Si .007%, S .011%.   The toroid made from it was annealed in the same way as described above.   The toroid, after annealing, was not examined at once microphotographically, and it was thought that it had remained very pure.   After the series of readings was taken with this specimen, the pure iron described in the last paragraph was examined to see whether the results would agree.   The results, however, came out widely different.   Micro-
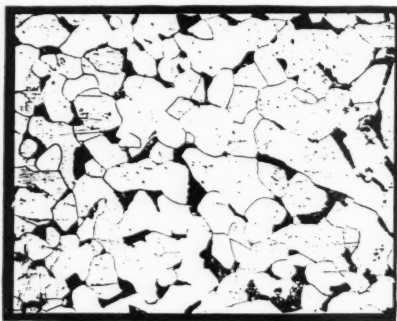
FIGURE 5.   Microphotograph of Slightly Carbonized Iron.

photographic examination of the specimen from Bureau of Standards then showed that it had been slightly carbonized while annealing. The results obtained with this specimen are therefore to be taken as results for slightly carbonized iron.   The difference between the specimen of pure iron and that of slightly carbonized iron is clearly seen in figures 4 to 5.   The microphotographs were taken with the two anchor rings themselves after their pressure coefficients had been determined.   An alcoholic solution of 5% nitric acid was used as the etching agent, with a magnification of 125 diameters.

After much work was done on the specimen from the Bureau of Standards, it was certainly a great disappointment to know that the specimen had been slightly carbonized.   The impressive lesson is that for any magnetic measurement of pure iron, a microphotograph should be taken before and after any heat treatment.

Steels.—Two kinds of steel, of carbon content .10% and .30% were examined. The method of annealing was the same as for iron.

Nickel.—The specimen of nickel examined had the following analysis: Approximately 99.1% Ni, 0.5% Co, together with small amounts of Fe, Si, C, and Cu. It was annealed by heating to 900° C., followed by slow cooling.

Cobalt.—The specimen was obtained from the Bureau of Standards which again obtained it from Kahlbaum. It came in the form of little cubes. A casting was made in a vacuum furnace. The toroid made from the casting was first annealed by heating to 1000° C. After a series of readings was taken, it was annealed again, using the method of Kalmus (9), i.e. keeping the specimen between 500° and 600° C. for several hours. The second series of readings, however, gave no different results.

In table I are given the dimensions of the toroids.

TABLE I.

Specimen	Mean diameter	Arm width	Arm width/mean diameter
pure iron	1.110 cm.	.294 cm.	.265
slightly carbonized iron	1.153	.300	.260
.10% C. steel	1.018	.317	.312
.30% C. steel	1.054	.305	.289
nickel	1.092	.292	.267
cobalt	1.067	.336	.312

These dimensions will be needed in applying a certain correction to our experimental results, to be explained later, as well as in calculating H and B.

EXPERIMENTAL RESULTS.

$\Delta B/B_0$ *at Room Temperature.*—Sample curves of constant field runs are given in figures 6 to 8. They show that, for iron, the decrease of magnetization is proportional to pressure within the limits of experimental error. The individual data for $H = 0.52$ and $H = 15.6$ are not as regular as for $H = 1.30$ and 4.69. The reason is that for $H = 0.52$ the total deflection is small, and for $H = 15.6$, the percentage change becomes small.

In the case of nickel, the increase of magnetization is also approximately linear with pressure. That the curve has a slightly increasing slope is explained by the fact that the specimen, though demagnetized before taking the series of readings for the curve, was not further
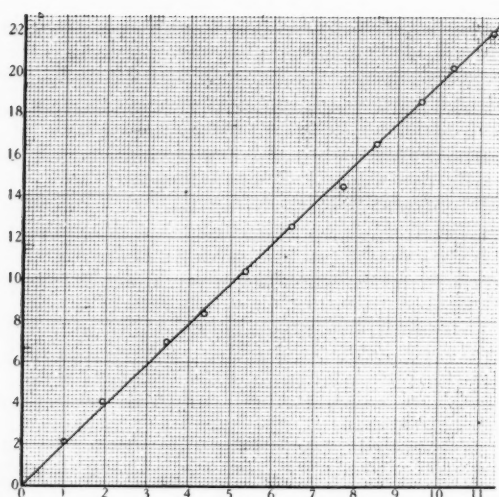
FIGURE 6.   Effect of pressure, at 20°, on the induction of slightly carbon-
ized iron, under a constant field of 1.30 Gauss.   Ordinates are $-(\Delta B/B_0)$
in per cent, abscissae pressure in thousands of kg./cm^2.

demagnetized in the course of increasing pressure.   (See the paragraph
on the effect of imperfect demagnetization).

Sample data sheets for constant field runs are given in tables II,

TABLE II.

CONSTANT FIELD RUN FOR SLIGHTLY CARBONIZED IRON AT $H = 1.30$

Pressure	Residual Rev. Defl.	$\Delta B/B_0$ (per cent)	
		Uncorrected	Corrected
0 kg./cm.^{2}	5.09 cm.	0	0
1010	4.72	− 2.17	− 2.15
1930	4.39	− 4.10	− 4.06
3470	3.89	− 7.04	− 6.97
4370	3.65	− 8.44	− 8.35
5360	3.30	− 10.49	− 10.38
6460	2.93	− 12.67	− 12.54
7590	2.60	− 14.60	− 14.45
8490	2.24	− 16.70	− 16.52
9570	1.89	− 18.76	− 18.56
10320	1.61	− 20.40	− 20.19
11290	1.32	− 22.08	− 21.85

data for fig. 6, and III, part of data for fig. 7. A sample data sheet for a constant pressure run is given in table IV, data for fig. 9.
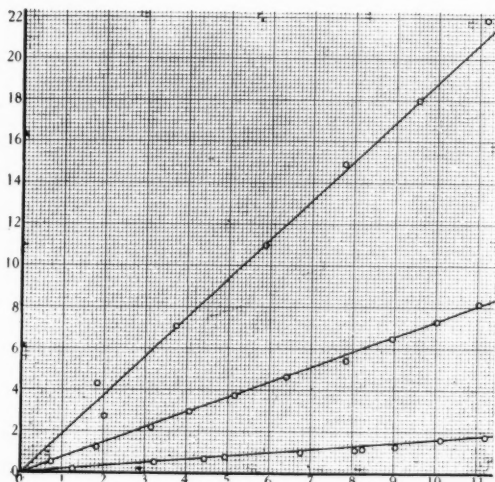
FIGURE 7.   Effect of pressure, at 20°, on the induction of slightly carbonized iron under constant fields of respectively 0.52, 4.69 and 15.6 Gauss, reading from the top down.   Ordinates are $-(\Delta B/B_0)$ in per cent, abscissae pressure in thousands of kg./cm².
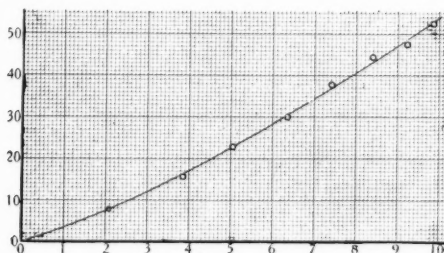

FIGURE 8.   Effect of pressure, at 20°, on the induction of nickel under a constant field of approximately 2 Gauss.   Ordinates are $+(\Delta B/B_0)$ in per cent, abscissae pressure in thousands of kg./cm².
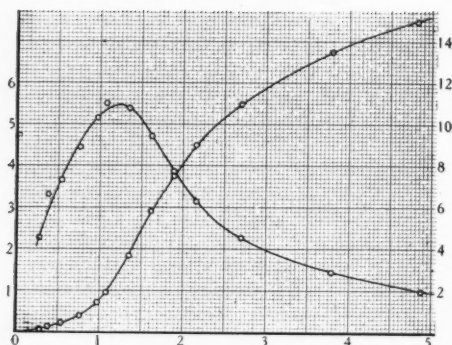
FIGURE 9. Results for pure iron at 20°. Abscissae are H, in Gauss. Ordinates, left hand scale, $-(\Delta B/B_0)$ in per cent per 1000 kg./cm.²; right hand scale, B in thousands of Gauss.

TABLE III.

CONSTANT FIELD RUN FOR SLIGHTLY CARBONIZED IRON AT $H = 0.52$

Pressure	Residual Rev. Defl.	$\Delta B/B_0$ (per cent) Uncorrected	Corrected
0	2.53		
1850	2.42	$-\ 4.4$	$-\ 4.3$
2020	2.46	$-\ 2.8$	$-\ 2.7$
3740	2.35	$-\ 7.1$	$-\ 7.1$
5880	2.25	-11.1	-11.0
7770	2.15	-15.0	-14.9
9540	2.07	-18.2	-18.0
11180	1.97	-22.2	-21.9

NOTE: The straight line passing through the origin and fitting well the points corresponding to high pressures passes between the points corresponding to 1850 and 2020 kg./cm.² The irregularity at low pressures is evidently due to difficulty of accurate measurement.

TABLE IV.

CONSTANT PRESSURE RUN FOR PURE IRON AT $P = 7510$ KG./CM.²

H	Total Rev. Defl. at $p = 0$	Total Rev. Defl. at $P = 7510$	$\Delta B/B_0$ (per cent) Uncorrected	Corrected	$\Delta B/B_0$ (per cent) for 1000 kg./cm.² Corrected
0.27	.48	.40	17	17	-2.27
0.38	.80	.60	25.0	24.8	-3.31
0.54	1.41	1.02	27.7	27.5	-3.67

TABLE IV.—*Continued.*

H	Total Rev. Defl. at $p = 0$	Total Rev. Defl. at $P = 7510$	$\Delta B/B_0$ (per cent) Uncorrected	$\Delta B/B_0$ (per cent) Corrected	$\Delta B/B_0$ (per cent) for 1000 kg./cm.^{2} Corrected
0.76	2.62	1.74	33.6	33.4	- 4.45
0.97	4.72	2.89	38.8	38.6	− 5.15
1.08	6.38	3.73	41.5	41.3	− 5.51
1.35	12.31	7.33	40.5	40.3	− 5.38
1.62	19.41	12.50	35.6	35 4	− 4.72
1.89	25.10	17.81	29.1	28.9	− 3.86
2.16	30.00	22.93	23.7	23.5	− 3.14
2.70	14.37*	30.29	17.2	17.0	− 2.27
3.79	17.63*	15.71*	10.9	10.7	− 1.43
4.87	19 58*	18.10*	7.57	7.42	− 0.99

The remaining data are given in Tables V to IX, and in figures 10

TABLE V.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR SLIGHTLY CARBONIZED IRON.

H	B	Pressure Coefficient
0.26	244	− 1.30
0.52	1048	− 1.88
0.62	1102	− 2.14
0.73	1566	− 2.36
0.78	1915	− 2.66
0.83	2170	− 2.43
0.93	2800	− 2.38
1.04	3640	− 2.19
1.30	5480	− 1.89
1.56	6020	− 1.71
1.82	7050	− 1.46
2.08	7620	− 1.27
4.69	11200	− 0.73
15.6	16580	− 0.16
52.0	18760	− 0.01

TABLE VI.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR .10% C STEEL.

H	B	Observed Pressure Coefficient
0.57	300	− 0.57
1.13	836	− 0.95
1.42	1292	− 1.28

* Constants of circuits changed.

TABLE VI.—*Continued.*

H	B	Observed Pressure Coefficient
1.70	1928	− 1.59
1.98	2695	− 1.69
2.26	3570	− 1.57
2.83	4870	− 1.52
3.40	6030	− 1.33
3.96	6890	− 1.15
4.53	7640	− 1.06
5.15	8390	− 0.97
5.66	8910	− 0.93
11.32	12210	− 0.58
16.98	13960	− 0.38
22.64	15000	− 0.25
28.30	15660	− 0.18
33.96	16170	− 0.10

TABLE VII.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR .30% C STEEL

H	B	Observed Pressure Coefficient
0.69	263	− 1.66
1.24	614	− 1.80
1.48	851	− 1.88
1.73	1163	− 1.99
1.98	1588	− 2.05
2.47	2565	− 2.30
2.96	3610	− 2.36
3.46	4560	− 2.16
3.96	5510	− 2.02
4.50	6440	− 1.84
4.94	7090	− 1.70

TABLE VIII.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR NICKEL.

H	B	Observed Pressure Coefficient
.223	33	+ 2.5
.446	76	+ 3.2
.624	118	+ 4.2
.892	210	+ 5.4
1.115	303	+ 5.56
1.338	402	+ 5.62
1.561	503	+ 5.34

TABLE VIII.—*Continued.*

H	B	Observed Pressure Coefficient
1.784	603	+ 5.22
2.01	692	+ 4.97
2.23	775	+ 4.76
4.46	1478	+ 4.57
6.69	2000	+ 3.87
8.92	·2480	+ 2.82
11.15	2910	+ 2.32
13.38	3300	+ 1.82
15.61	3640	+ 1.37
17.84	3930	+ 1.10
20.07	4170	+ 0.78
22.30	4360	+ 0.71

TABLE IX.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR COBALT.

H	B	Observed Pressure Coefficient
10.9	380	− 0.2
16.3	668	− 0.2
21.8	981	− 0.2
27.3	1278	− 0.2
32.8	1562	+ 0.090
41.0	1942	+ 0.39
45.2	2130	+ 0.27
48.0	2250	+ 0.26
56.1	2580	+ 0.25
64.3	2885	+ 0.30
72.4	3165	+ 0.38
79.5	3420	+ 0.41

to 14; the pressure coefficients are given as before as percentage changes per 1000 kg./cm². The numbers are obtained from the slope of the best fitting straight line in the case of the constant field runs. When pressure coefficients are deduced from constant pressure runs, use is made of the linear relation between percentage change and pressure established by the constant field runs. On account of the fact that our toroids were not thin, the pressure coefficient thus reported is a sort of average pressure coefficient corresponding to some average field. To reduce these experimental values to true coefficients corresponding to definite values of field involves the solution of an integral equation. The writer had not been able to solve the

integral equation, but believes that the data presented in this paper, namely, the B-H curves, the observed pressure coefficients, and the dimensions of the specimens, are sufficient for reducing the experimental values to true coefficients when the method of solving the integral equation is discovered.
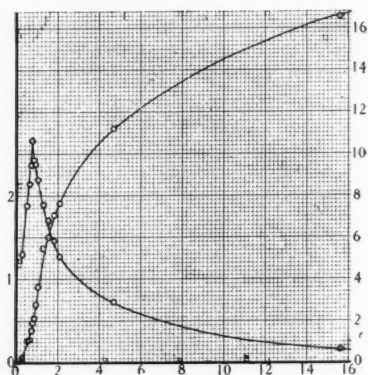

FIGURE 10. Results for slightly carbonized iron at 20°. Abscissae, H in Gauss. Ordinates, left hand scale, $-(\Delta B/B_0)$ in per cent per 1000 kg./cm.^{2}; right hand scale, B in thousands of Gauss.

FIGURE 11. Results for .10% carbon steel at 20°. Abscissae, H in Gauss. Ordinates, left hand scale, $-(\Delta B/B_0)$ in per cent per 1000 kg./cm.^{2}; right hand scale, B in thousands of Gauss.
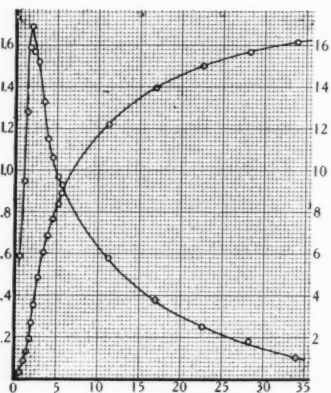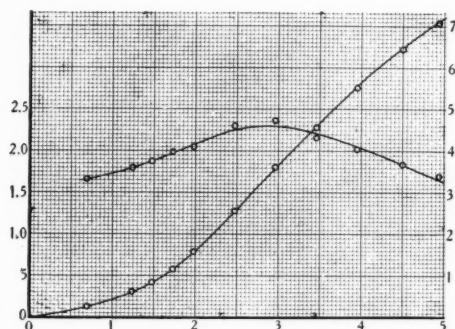
FIGURE 12. Results for .30% carbon steel at 20°. Abscissae, H in Gauss. Ordinates, left hand scale, $-(\Delta B/B_0)$ in per cent per 1000 kg./cm.²; right hand scale, B in thousands of Gauss.
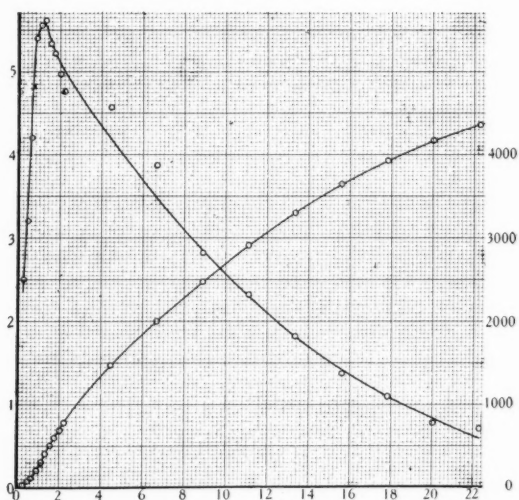

FIG. 13. Results for nickel at 20°. Abscissae, H in Gauss. Ordinates, left hand scale, $+(\Delta B/B_0)$ in per cent per 1000 kg./cm.²; right hand scale, B in thousands of Gauss.
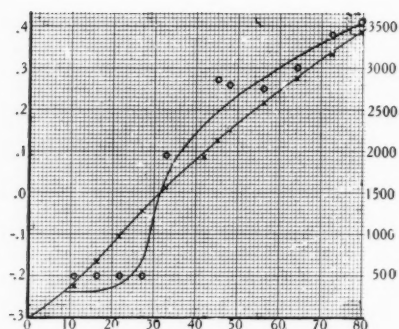
FIGURE 14.   Results for cobalt at 20°.   Abscissae, H in Gauss.   Ordinates left hand scale, $\Delta B/B_0$ in per cent per 1000 kg./cm.^{2} (circles); right hand scale, B in Gauss (crosses).

The Effect of Pressure on the Retentivity of Pure Iron.—From the fact that pressure is able to set free the residual magnetism of iron, it is expected that the retentivity of iron will decrease under pressure.   By retentivity as measured here, we mean, when expressed in per cent:—

$$\frac{100 \ (\tfrac{1}{2} \ \text{Rev. Defl.—withdrawal deflection})}{\tfrac{1}{2} \ \text{Rev. Defl.}}$$

The withdrawal deflection is obtained by opening the primary circuit. Two kinds of withdrawal deflection are recorded below: one is obtained after many reversals, and the other is obtained after applying and withdrawing the field in one direction several times.   We shall call the former cyclic withdrawal deflection, and the retentivity computed from it cyclic retentivity; the latter will be called the minimum withdrawal deflection, and the retentivity computed from it maximum retentivity.

The following data are obtained at $H = 1.62$.

	$P = 0$	$P = 7600$	$P = 8630$	$P = 10340$
1.2 Rev. Defl.	9.74	6.21	2.92	2.46
Cyclic withdrawal defl.	2.36	2.04	1.88	1.68
Min. withdrawal defl.	1.97	1.79	1.79	1.65
Cyclic Retentivity	75.8%	67.1	35.6	31.7
Max. Retentivity	79.8	71.2	38.7	33.0

The run was undertaken merely to obtain some idea of the effect, so only a few points were taken. For some reason the point at $P = 7600$ is bad; there seems to be no reason why there should be a sharp drop of retentivity around 8000 kg./cm². ·

The Temperature Coefficient of the Pressure Coefficient of Magnetization.—With the data of four magnetization curves, the following four coefficients can be obtained:

(1) temperature coefficient of magnetization under no pressure
(2)   "   "   "   "   under pressure
(3) pressure   "   "   "   at room temperature
(4)   "   "   "   "   at 100° C.

As before, by pressure coefficient we shall mean percentage change per 1000 kg./cm². By temperature coefficient we shall mean the average percentage change per degree averaged over an interval of 75°, from 25° to 100° C. In Table X are presented such a set of coef-

TABLE X.

TEMPERATURE AND PRESSURE COEFFICIENTS OF MAGNETIZATION OF NICKEL

	Temp. Coef.		Pressure Coef.	
H	At $P = 0$	At $P = 7300$	At 24°.8	At 99°.8
.223	.57% per degree	.647	2.5% per 1000 kg./cm.²	3.1
.446	.595	.618	3.2	3.48
.624	.587	.472	4.2	4.6
.892	.427	.199	5.4	3.04
1.115	.32	.1	5.56	3.06
1.338	.238	− 0.053	5.62	2.02
1.561	.172	− 0.108	5.34	1.79
1.784	.143	− 0.138	5.22	1.59
2.01	.12	− 0.147	4.97	1.53
2.23	.099	− 0.157	4.76	1.44
4.46	.069			0.63
8.92	.039			0.52

ficients for nickel. In calculating the temperature coefficient under a certain pressure, the value of B at 25° C. and under that pressure is used as the zero level against which percentage changes are figured; in calculating the pressure coefficient at a certain temperature, the value of B under no pressure and at that temperature is used as the zero level.

In looking over the table, we are first impressed with the fact that, for nickel, pressure has a large effect on the temperature coefficient. While under no pressure the familiar reversal from positive to negative temperature coefficient does not occur even at $H = 8.92$; it occurs under $P = 7300$ at $H = 1.26$. To confirm these results, two constant field runs at $H = 2.01$ were made:—the run under no pressure gave the temperature coefficient $+ .118\%$ per degree and that under 8080 kg./cm.^{2} gave $- .170\%$ per degree. On the basis of the coefficient $- 0.147$ for the same field under 7300 (see Table X) the coefficient under 8080 would be $- .176$. The agreement is fair.

We see that at fields greater than .5 pressure has the effect of decreasing algebraically the temperature coefficient. Since the pressure coefficient is positive for nickel, it is mathematically necessary that temperature has the effect of decreasing the pressure coefficient. This fact is shown in the last two columns of Table X. The reader will also note that $d^2B/dpdt$ is positive at very low fields.

For pure iron, it was found that pressure has the effect of increasing algebraically the temperature coefficient. Since the pressure coefficient is negative for iron, it follows mathematically that temperature has the effect of increasing algebraically the pressure coefficient (i.e. decreasing the absolute value). The data of Table XI illustrate these facts.

TABLE XI.

TEMPERATURE AND PRESSURE COEFFICIENTS OF MAGNETIZATION OF PURE IRON.

H	Temp. Coef. Under no Pressure	Under $P = 7050$	Pressure Coef. At 25° C.	At 100° C.
.43	$+ .147$	$+ .177$	$- 3.25$	$- 0.99$
.61	.171	.195	$- 3.93$	$- 2.09$
.86	.232	.247	$- 4.80$	$- 3.60$
1.08	.285	.294	$- 5.48$	$- 4.87$
1.30	.263	.264	$- 5.51$	$- 5.39$
1.51	.153	.158	$- 4.97$	$- 4.65$
1.73	.126	.135	$- 4.36$	$- 3.70$
1.94	.097	.106	$- 3.79$	$- 3.08$
2.16	.072	.079	$- 3.14$	$- 2.61$
4.32			$- 1.20$	$- 1.23$

Summing up, we see that the crossed derivative $d^2B/dpdt$ is preeminently negative for nickel but positive for iron. The facts seem to indicate that the distance between atoms or molecules is one of the chief factors that control the magnetostriction phenomena. Regard-

less of the sign of the pressure coefficient, temperature has the effect of decreasing numerically the pressure coefficient.   That is, under the same pressure and the same field, the change of magnetization is less at a higher temperature than at a lower temperature.   The underlying reason seems to be that on account of the thermal expansion of the atomic lattice, the same amount of pressure is less effective in producing change of magnetization.

DISCUSSION OF THE EXPERIMENTAL RESULTS.

For all the fields investigated, the pressure coefficient of magnetization is negative for the different varieties of iron and steel, but positive for nickel.   For cobalt, the pressure coefficient is negative, that is, iron-like for fields below about 30 gausses; for higher fields, the pressure coefficient becomes positive, that is nickel-like.   The intermediate character of the results for cobalt is very interesting and must have theoretical significance in view of the fact that cobalt is also between iron and nickel when the elements are arranged in the order of their atomic numbers.

When we plot the absolute value of the pressure coefficient against field, the curves for iron, steel, and nickel all show a rather sharp maximum at a certain field.   For pure iron, the maximum percentage change is -5.5 per 1000 kg./cm.^{2}, occurring at $H = 1.2$; for nickel, the maximum is $+5.6$, at $H = 1.3$.

It may be asked whether the percentage change also vanishes with a vanishing field.   To answer this question experimentally would be a fussy undertaking, on account of the small value of B at very low fields, and consequently, the necessity of winding a large number of secondary turns.   From the fact that both permeability and differential permeability (i.e. the derivative dB/dH) are not zero for $H = 0$, it is highly probable that the per cent change also does not vanish for $H = 0$.

With regard to iron, steel and nickel, another interesting question is whether the sign of the pressure coefficient would reverse at very high fields.   The question is especially important for iron and steel which exhibit reversal points for both the Joule effect and the Villari effect.   For the slightly carbonized iron, at $H = 52$, the percentage decrease was found to be slightly less than .01% per 1000 kg./cm.^{2}.   A test was also made at $H = 112$, with pressure put on and released several times.   The effect of pressure is unmistakably a decrease of permeability, however minute that may be.   The data are, however,

still insufficient to answer the question definitely. All we can con-
clude from the tendencies of the curves and the phenomena of mag-
netic saturation is that for very high fields, the pressure coefficient of
magnetization assumes a very small value, either positive or negative,
which is perhaps of the order of the compressibility of metals. The
question is fundamentally important and worthy of further attack,
either theoretical or experimental. If the limiting value of the pres-
sure coefficient for very high fields were rigorously zero, the saturation
intensity of magnetization per unit volume would be independent of
pressure, when the change of dimensions due to pressure is taken into
account in calculating B and H. Assuming that the atoms are the
magnetons, then there would be a decrease of the magnetic moment of
the atom under pressure. A conclusion of this sort would involve
very important consequences. Incidentally, the reader may note
that there is also no experimental data on such a problem as the
saturation intensity of an iron rod under tension.

With regard to iron, the data of the present research definitely
contradict the positive pressure coefficient of magnetization obtained
by Miss Frisbie at low fields. In the earlier course of the work,
what Miss Frisbie observed was indeed also observed by the author.
The cause was later traced to imperfect demagnetization, the pressure
being able to set free a part of the residual magnetism of iron. When
demagnetization was perfect, the erroneous effect also disappeared.
The reversal obtained by Miss Frisbie might also be due to imperfect
demagnetization.

THEORETICAL CONSIDERATIONS.

Thermodynamically, the effect of hydrostatic pressure on mag-
netization is reciprocally connected with the volume change produced
by magnetization, so that from the results of the present research,
conclusions with regard to the latter effect can be deduced with a
reasonable degree of certainty. But the reader must carefully note
that we cannot draw any necessary conclusion in regard to the other
types of magnetostriction phenomena. The following classification
clearly illustrates the meaning of this statement:—

A. Effect of stress on magnetization
   1. Effect of pressure (hydro-
      static) on magnetization
   2. Effect of tension on mag-
      netization (Villari effect)

B. Strain due to magnetization
   1. Volume change due to
      magnetization
   2. Length change due to
      magnetization (Joule ef-
      fect)

3. Reciprocals of Wiedemann effect   3. Wiedemann effect
   (a) Transient current effect
   (b) Longitudinal magnetization-
      effect

This classification presents clearly the theoretical connection between the various phenomena. Those in the same horizontal line are reciprocals of each other and are connected by thermodynamics; while those in the same column are connected by mechanisms deeply rooted in the nature of elasticity and magnetism.

The phenomena in the above list are quite complicated. They depend not only on the nature of the magnetized substance, but also on the geometrical form of the specimen. The description of these phenomena is out of place here. In view of the confusion that exists in the literature with regard to the theories of magnetostriction, it will be worth while to outline the general theoretical methods.

There are two methods of treating magnetostriction. The first method may be called the elasticity method. It consists of first finding the force per unit volume acting on the matter when placed in a magnetic field. This is accomplished by equating the change of electromagnetic energy with the work of virtual displacement of the parts against the force. The expression for the body force, in the limit at surfaces of discontinuity, then gives an expression for the surface force acting at the boundary between the two media. The elastic stresses and the strains are then computed by the equations of elasticity, so as to equilibrate the given body and surface forces.

The other method may be called the energy method or the thermodynamic method. The only physical part of the reasoning consists in setting up an expression either for the heat absorbed or for the work done by the matter. All the rest follows the formal work of thermodynamics, assuming reversibility.

Let us now describe the elasticity method more in detail. Let F be the vector force per unit volume acting on matter when placed in the magnetic field. Let X_x, Y_y, Z_z, $X_y = Y_x$, $Y_z = Z_y$, $Z_x = X_z$ be the components of the elastic stress. Then the equations of equilibrium state:

$$\frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} = -F_x$$

$$\frac{\partial Y_x}{\partial x} + \frac{\partial Y_y}{\partial y} + \frac{\partial Y_z}{\partial z} = -F_y \qquad (1)$$

$$\frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y} + \frac{\partial Z_z}{\partial z} = -F_z$$

The strains are then computed from the stresses by the familiar relations of elasticity involving the elastic constants.

In the literature, another set of stresses has been introduced, namely, that in the ether. We will denote the components of this set by A_x, B_y, C_z, $A_y = B_x$, $B_z = C_y$, $C_x = A_z$. The assumption made by Maxwell and others (6) is simply that

$$X_x = -A_x, \qquad Y_y = -B_y, \qquad Z_z = -C_z$$
$$X_y = -A_y, \qquad Y_z = -B_z, \qquad Z_z = -C_x \tag{3}$$

This assumption is however not compelling. In the following treatment we will make no use of the stresses in the ether.

With the thermodynamic method, the work amounts to applying, in many cases, only a single general theorem. For an isothermal reversible process, from the two laws of thermodynamics, we know that the work term dA is a total differential. Let $dA = A_\phi d\phi + A_\psi d\psi$, where ϕ and ψ are any two variables that may enter our problem, and $A\phi$ and $A\psi$ are the corresponding coefficients.

If ϕ and ψ are also independent variables, then

$$\left(\frac{\partial A}{\partial \phi} \right)_\psi = A_\phi, \qquad \left(\frac{\partial A}{\partial \psi} \right)_\phi = A_\psi \tag{4}$$

We have therefore

$$\left(\frac{\partial A_\phi}{\partial \psi} \right)_\phi = \left(\frac{\partial A_\psi}{\partial \phi} \right)_\psi$$

The simplest application of (4) concerns the reciprocal relation between the volume change produced by magnetization and the change of magnetization due to pressure. Let V and I be the total volume and the total magnetic moment of a substance under hydrostatic pressure. There will be not only a change of volume due to pressure, but also a change of magnetization due to pressure. The work term consists of two parts: (1) work done by the substance when its volume increases by dV, (2) work done upon the substance when its magnetization increases by dI. $\therefore dA = -HdI + pdV$.

When we take p and H as independent variables, then

$$dA = \left(-H\frac{\partial I}{\partial p} + p\frac{\partial V}{\partial p} \right)\partial p + \left(-H\frac{\partial I}{\partial H} + p\frac{\partial V}{\partial H} \right)dH$$

Whence

$$-\left(\frac{\partial I}{\partial p} \right)_H = \left(\frac{\partial V}{\partial H} \right)_p \qquad \text{by (4)} \tag{6}$$

It is important to note that in this equation I and V mean the total magnetic moment and the total volume of the specimen, not per unit volume or per gram. We now let $I = iV$, where i = intensity of magnetization. Then

$$\left(\frac{\partial V}{\partial H}\right)_p = -V\left(\frac{\partial i}{\partial p}\right)_H - i\left(\frac{\partial V}{\partial p}\right)_H$$

Dividing by V_0, since V/V_0 is practically unity, we have

$$\frac{1}{V_0}\left(\frac{\partial V}{\partial H}\right)_p = -\left(\frac{\partial i}{\partial p}\right)_H - i\frac{1}{V_0}\left(\frac{\partial V}{\partial p}\right)_H \tag{7}$$

$(\partial i/\partial p)_H$ is the pressure coefficient reported in this paper.

In this deduction, we make no specifications about the form of specimen, so equation (7) is true for any form of specimen provided there is no end effect. This condition is satisfied in the case of a thin toroid.

Let us now see whether equation (7) is in agreement with results obtained in the other way. Since

$$-\frac{1}{V_0}\left(\frac{\partial V}{\partial p}\right)_H = \frac{3}{E}\left(3 - \frac{E}{K}\right) = \frac{3}{E}(1 - 2\sigma)$$

where E is Young's modulus, K, rigidity; and σ, Poisson's ratio, assuming that $(1/V_0)(\partial V/\partial p)$ changes little with magnetization, we obtain from (7) by integrating with respect to H,

$$\left(\frac{\Delta V}{V_0}\right)_p = \frac{3}{E}(1 - 2\sigma)\int_0^H i\,dH - \int_0^H \frac{1}{i_0}\left(\frac{\partial i}{\partial p}\right)_H i_0\,dH \tag{8}$$

This is in agreement with what was obtained by Kolacek by the elasticity method, since the expression $(\partial i/\partial P) + (\partial i/\partial Q) + (\partial i/\partial R)$ in his formula, where P, Q, R are the principal tensile stresses, is exactly our $\partial i/\partial p$. With (8), we analyse the volume change produced by magnetization into two parts: one part is intimately connected with the pressure coefficient of magnetization, and the other part is still present even when the pressure coefficient is zero. At low fields, the part connected with the pressure coefficient is more important than the other part; but for high fields, the latter becomes increasingly important, since $(1/i_0)(\partial i/\partial p)$ approaches zero asymptotically for high fields.

On the basis of the data of this paper, the volume changes produced

by magnetization at fields below 100 gausses can be calculated. But to extend the calculation to much higher fields, further assumptions are necessary.

For iron, whose pressure coefficient of magnetization is negative, both terms work toward an increase of volume. $1/i_0(\partial i/\partial p)_H i_0$ is of the order of 10^{-8}, $3(1 - 2\sigma)i/E$ is of the order of 10^{-9} for low fields. When the contribution by $\partial i/\partial p$ becomes inappreciable, say after saturation, the volume still increases at the rate of $3(1 - 2\sigma)i$ saturation$/E$ per gauss. Taking $\sigma = \frac{1}{4}$, $E = 2 \times 10^{12}$, $i_{sat} = 2 \times 10^3$, this amounts to about 1.5×10^{-9} per gauss.

For nickel, the two terms work in opposite directions. The term connected with $\partial i/\partial p$ will give a decrease of volume. This decrease will become approximately constant when the field is sufficiently high; after that, the volume will continually increase at the small rate of about $.4 \times 10^{-9}$ per gauss, so that the total change of volume eventually passes through zero, and at still higher fields becomes increasingly positive.

For cobalt, there will be an initial increase of volume at low fields, probably too small to be observed. The further course of the volume change will be similar to the case of nickel, being at first a decrease, then a reversal of direction, passing through zero, and eventually becoming positive. Since cobalt has a much greater saturation intensity of magnetization than nickel, the increase of volume at very high fields will be more pronounced in the case of cobalt, and the volume will pass through its initial value after the early contraction at a smaller field.

These qualitative conclusions are in agreement with the facts and the tendencies observed by Nagaoka and Honda (8). The initial volume increase of cobalt has so far not been observed.

Equation (8) gives the volume change produced by magnetization under any pressure. When the i in the first term on the right hand side is i_0, we obtain the volume change produced by magnetization under that pressure. By differentiating (8), we can obtain the pressure coefficient of magnetostriction.

To explain the effect of pressure on magnetization on the basis of the electron theory is not an easy matter. In view of the fact that we yet have no adequate theory of ferromagnetism, it would seem to be premature to attempt any complete explanation of the pressure effect on magnetization. According to current theories, the explanation of ferromagnetism involves at least the following considerations:—

1. The existence of saturation intensity points clearly to the existence of some sort of elementary magnet.

2. Orientation of these elementary magnets must be the principal factor to account for the magnetization curve.

3. The important influences that work against the orienting effect of the external field are thermal agitation and the mutual action between the elementary magnets.

4. It is found that thermal agitation and the orienting effect of the external field are insufficient to explain ferromagnetism. The high permeability of the ferromagnetic substances points towards the existence of some intense molecular field. The nature of this molecular field is yet unknown.

5. The nature of the molecular field presumably depends upon the structure of the atomic lattice and the shape of the atom.

It is this last point that has most intimate connection with the effect we have studied. Conceivably, the existence of both positive and negative pressure coefficients of magnetization is to be explained by the possibility that pressure may either increase or decrease the magnitude of the molecular field, according to differences of its structure. It is interesting to note that while on the one hand we have the characteristic difference between iron and nickel with regard to the pressure coefficient of magnetization, on the other hand we also know that these two metals have different types of crystal structure, iron in the α range being body-centered cubic, nickel being face-centered cubic. Although there is no correlation between ferromagnetism and the type of crystal lattice, it is still quite possible that the characteristic differences in the crystal structure and in the pressure coefficient of magnetization might be consequences of the same primary cause. Attempts to explain ferromagnetism with the molecular field of elementary magnets are so far quantitatively unsatisfactory. The complications in the magnetic behavior of the chemical compounds seem to indicate that the valence electron must play a very important rôle in the ferromagnetic phenomena. This factor, so far, has not been considered in any theory of ferromagnetism. Iron, cobalt, and nickel mark the end of a period, the elements of which, according to Bohr's recent views, suffer a gradual transformation in the inner configuration of the atom. The kernel of the atom is therefore characterized by greater asymmetry. How does this asymmetry influence the valence electron? How does the valence electron influence ferromagnetism? These are interesting problems.

SUMMARY.

1. Within the pressure range 0–12000 kg./cm.^{2}, the change of magnetization at constant H is linear with pressure for iron, cobalt, and nickel.

2. Within the field range 0–100 gauss, the pressure coefficient of magnetization per unit volume is negative for iron, but positive for nickel. It is highly probable on experimental grounds that the sign of the coefficient will not reverse for higher fields.

3. For cobalt, the pressure coefficient of magnetization per unit volume is negative for fields below about 30 gausses, but positive for higher fields.

4. For pure iron at room temperature the percentage change of magnetization has a maximum of -5.5% per 1000 kg./cm.^{2} at H about 1.2. The percentage change decreases quite rapidly on both sides of this maximum, approaching the axis asymptotically for large H. Such a maximum change also exists for nickel, of value $+4.5\%$ per 1000 kg./cm.^{2} at $H = 1.3$.

5. For both pure iron and nickel, (except nickel in very low fields) the absolute value of the pressure coefficient of magnetization per unit volume is less at a higher temperature.

6. For iron, the percentage change of magnetization is a very sensitive function of its carbon content.

7. The retentivity of iron decreases under pressure.

8. By simple thermodynamics, it is shown that the volume change produced by magnetization can be analysed into two terms; the term involving the pressure coefficient of magnetization is important at low fields, while the other term not involving the pressure coefficient becomes increasingly important at high fields.

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